A new mathematical wake model for yawed wind turbines

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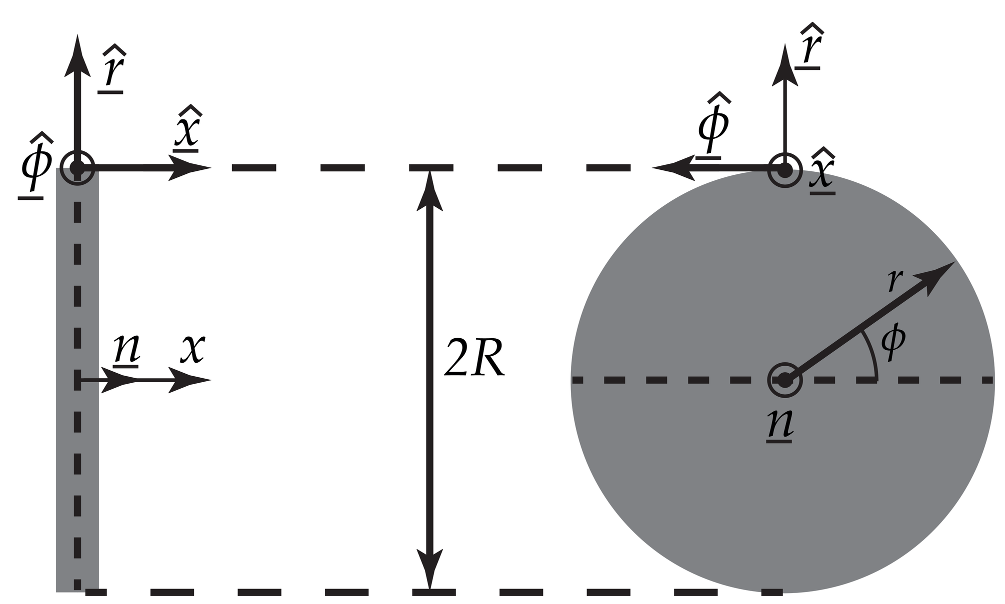
ABSTRACT

This study is aimed to provide an approximate solution of the wind flow when passes through a yawed wind turbine; and as a result, an estimate of its power production.

A mathematical model is directly deduced from the general conservation principles to describe the velocity field and pressure distribution under certain reasonable assumptions. The main equations that govern the flow will be carefully deduced one at a time according to some hypotheses.

This model aims to be fast and accurate enough to calculate a suitable estimate for the production of an entire wind farm. The idea of the actuator-disk D (see below the scheme) will be incorporated to model the wind turbine rotor, that can be subjected to the most general loading state. The accuracy and the validity of this model will be verified with on-field measurements.



MATHEMATICAL MODEL

The velocity deficit $\underline{v}'$ due to N_T wind turbines, each one at position x_k is obtained from the following PDE:

$$\left[(\underline{V}+\underline{v}')\cdot\underline{\mathrm{grad}}\right]\underline{v}' - \frac{1}{\mathcal{R}_t}\triangle\underline{v}' = -\sum_{k=1}^{N_t}\mathcal{E}_k\,\underline{\mathrm{grad}}\,p'_k \qquad (1)$$

Where p'_k is the pressure distribution due to the k-th wind turbine, given by the following equation centred at $\underline{x}_k$:

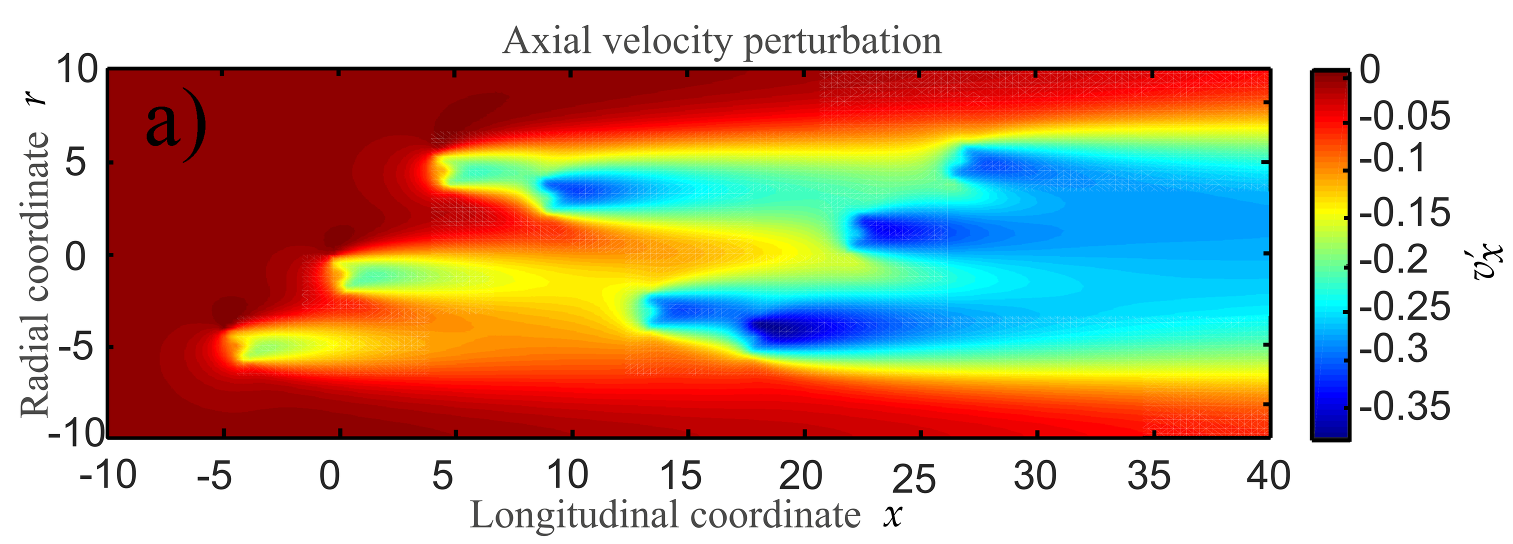
$$\triangle p'_k = 0 \qquad (2)$$

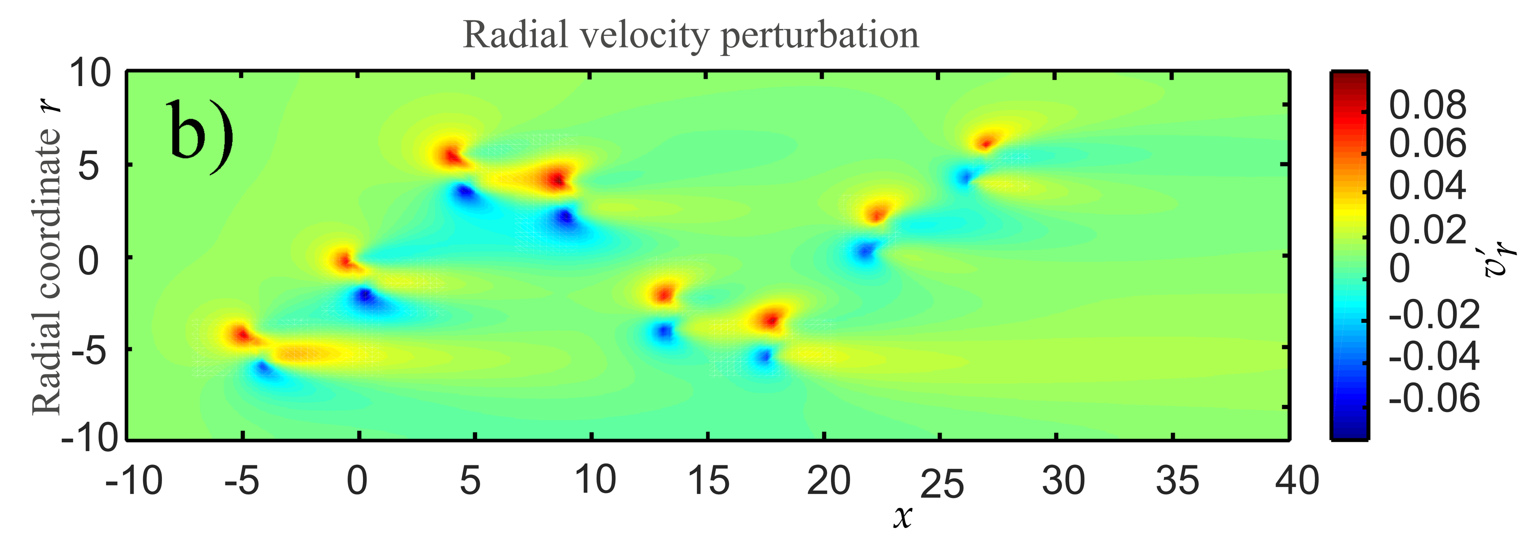
Subjected to the following boundary conditions:

$$\lim_{|\underline{x}|\to\infty}\underline{v}' = \underline{0} \qquad \lim_{|\underline{v}|\to\infty} p'_k = 0 \qquad [\![p'_k]\!]_D = \Delta p$$

RESULTS

The equation (2) was analytically solved and included in equation (1) as a source term. The pressure drop across the disk was chosen such that $\Delta p \propto (1+r)\sqrt{1-r^2}$. The simulations depicted in figures a) and b) ran with $\mathcal{E}_k = (\underline{v}\cdot\underline{n}_k)/3$ and $\mathcal{R}_t = 7$ using $\underline{V} = \underline{e}_x$.





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